1.1. Interference of Light

Objectives, Applications and Importance:

- Interference is used to determine the wavelength of light.
- It is used to find the difference in wavelengths of two spectral lines having small separation.
- It is used to find the thickness of transparent material.
- It is used to determine the refractive index of transparent solids, liquids and gases.
- It is used to test the optical flatness or planeness of the surfaces.
- It is used to find the reflecting power of lenses and prism.
- It is used to determine the radius of curvature of given Plano-convex lens and thickness of very thin objects such as piece of paper, hair etc.

Engineering applications of Interference:

- Optical interference is used for the antireflection coatings on complicated lenses.
- Optical filters with narrow band pass or band reject properties also operate by optical interference. These optical filters are often used in slide projectors to direct the heat produced that might damage film in transparencies away while sending the visible light through to produce the projected image.
- Interference is used in interference auto compensators in measurement engineering.
- Interference in thin films is used in non-reflecting coatings in engineering applications.
- Interference in thin films concept is used in non-reflecting coatings in engineering applications.

Principle of Superposition of waves:

It states that when a particle of a medium is simultaneously acted upon by two or more waves then the resultant displacement of a particle of the medium is the algebraic sum of the displacements of the same particle due to individual waves in the absence of one another.

$$y = y_1 + y_2$$

Here, y is the resultant displacement, y_1 and y_2 are the displacements of the particle due to individual waves.

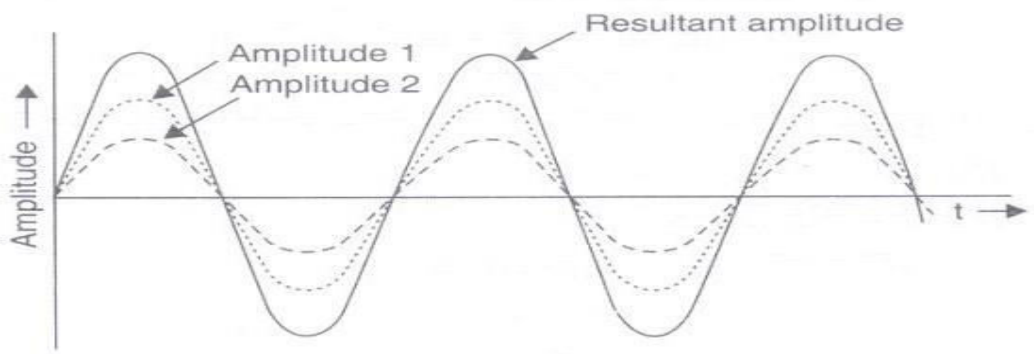
Superposition of waves leads to different phenomenon.

Examples of superposition of waves:

- Two waves of same frequency and same amplitude travelling in same direction are superimposed over each other gives rise to the phenomenon of ***interference***. (All waves)
- Two waves of slightly different frequencies and same amplitude travelling same direction superimposed over each other produces the phenomenon of ***beats. (Sound waves)***
- Two waves of same frequency and same amplitude moving in opposite direction superimposed over each other ***stationary waves*** are observed. (water waves)

1. Superposition of waves of Equal Phase and same Frequency:

Let us assume that two sinusoidal waves of the same frequency are traveling together in medium. The amplitude of first and second wave is A_1 and A_2 respectively. The waves have the same phase i.e. phase difference between them is zero. Then the crest of one wave falls exactly on the crest of the other wave and so do the trough (as shown in figure). According to the principle of superposition of waves, the resultant amplitude (A) is the sum of the individual amplitudes.



$A=A_1+A_2$

The resultant intensity (I) is the square of the sum of the amplitudes.

Resultant Intensity $I=A^2= (A_1+A_2)^2$

If much number of waves is traveling together in medium having amplitudes A_1, A_2, and A3……..

The resultant amplitude is $A=A_1+A_2+A_3$…………

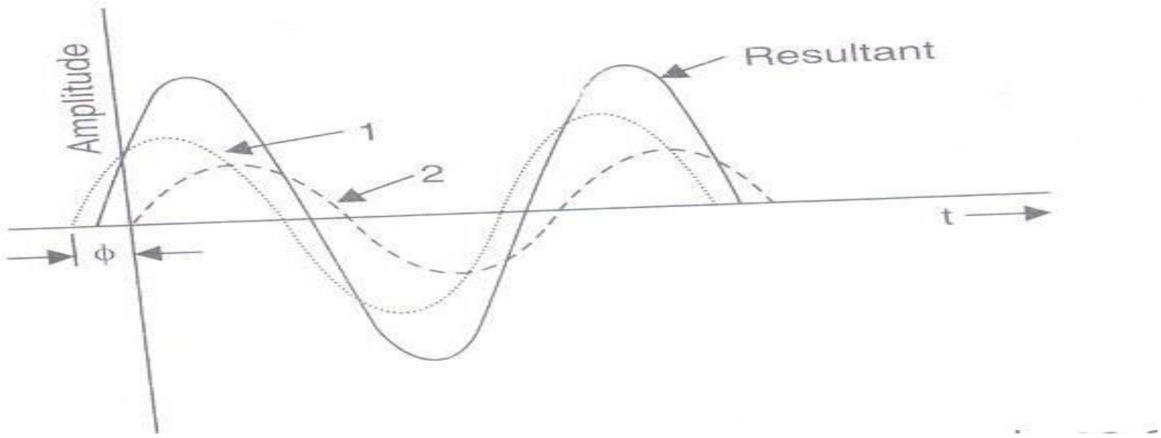
The resultant Intensity is $I= A^2= (A_1+ A_2+A_3\ldots\ldots\ldots\ldots)^2$

2. Superposition of waves of same Frequency and Constant Phase difference:

Let us consider two waves that have the same frequency but have a certain constant phase difference δ between them. In this case the crest of one wave does not exactly coincide with the crest of the other wave as shown in figure.

The two waves having the same frequency and constant phase difference δ can be represented by the equations

$$y_1 = a_1 \sin \omega t$$
$$y_2 = a_2 \sin(\omega t + \delta) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$



y_1 and y_2 are displacements of two waves, a_1 and a_2 are the amplitudes, δ is the constant phase difference and ω is the angular frequency of the waves. According to superposition principle, the resultant displacement is given by,

$$\begin{aligned} y &= y_1 + y_2 \\ &= a_1 \sin \omega t + a_2 \sin(\omega t + \delta) \\ &= a_1 \sin \omega t + a_2 (\sin \omega t \cos \delta + \cos \omega t \sin \delta) \\ &= (a_1 + a_2 \cos \delta) \sin \omega t + a_2 \cos \omega t \sin \delta \end{aligned}$$

Let

$$a_1 + a_2 \cos \delta = a \cos \phi \ldots.(1)$$
$$a_2 \sin \delta = a \sin \phi \ldots..(2)$$

Thus,

$$\begin{aligned} y &= a \cos \phi \sin \omega t + a \sin \phi \cos \omega t \\ &= a \sin(\omega t + \phi) \ldots\ldots.(3) \end{aligned}$$

This is the equation of resultant wave (simple harmonic motion) with amplitude a and phase angle ϕ.

Dividing equation (2) by equation (1),

$$\frac{a\sin\phi}{a\cos\phi} = \frac{a_2 \sin\delta}{a_1 + a_2\cos\delta}$$

$$\tan\phi = \frac{a_2 \sin\delta}{a_1 + a_2\cos\delta}$$

$$\phi = \tan^{-1}\left(\frac{a_2 \sin\delta}{a_1 + a_2\cos\delta}\right).........(4)$$

The phase angle of the resultant wave is given by the equation (4).

Squaring and adding the equations (1) and (2),

$$a^2\sin^2\phi + a^2\cos^2\phi = (a_1 + a_2\cos\delta)^2 + a_2^2\sin^2\delta$$

$$a^2(\sin^2\phi + \cos^2\phi) = a_1^2 + a_2^2\cos^2\delta + 2a_1a_2\cos\delta + a_2^2\sin^2\delta$$

$$a^2(1) = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\delta}............(5)$$

The resultant amplitude is given by equation (5).

Condition for maximum amplitude:

For a to be maximum,

$$\cos\delta = 1$$

$$\delta = 2n\pi$$

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\delta}.$$

$$= \sqrt{a_1^2 + a_2^2 + 2a_1a_2}.$$

$$= \sqrt{(a_1 + a_2)^2}$$

$$a = a_1 + a_2$$

When the amplitude is maximum, bright fringes are formed as a result of constructive interference.

$$\text{Path difference (P)} = \frac{\lambda}{2\pi}(\text{phase difference})$$

$$= \frac{\lambda}{2\pi}2n\pi$$

$$P = n\lambda$$

Condition for minimum amplitude:

For a to be minimum,

$$\cos\delta = -1$$

$$\delta = (2n+1)\pi$$

$$a = \sqrt{a_1^2 + a_2^2 - 2a_1a_2\cos\delta}.$$

$$= \sqrt{a_1^2 + a_2^2 - 2a_1a_2}.$$

$$= \sqrt{(a_1 - a_2)^2}$$

$$a = a_1 - a_2$$

When the amplitude is minimum, dark fringes are formed as a result of destructive interference.

Path difference (P) = $\frac{\lambda}{2\pi}\left(\text{phase difference}\right)$

$$P = \frac{\lambda}{2\pi}(2n+1)\pi$$

$$P = (2n+1)\frac{\lambda}{2}$$

Relation between Phase difference and path difference:

When the path difference between the waves is λ, then the phase difference is 2π.

For the path difference of x, the phase difference is $\frac{2\pi x}{\lambda}$.

Phase difference $\delta = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda}\left(\text{path difference}\right)$

Interference of light:

When two waves of same frequency and amplitude moving in same direction superimpose over each other the phenomenon is called interference. When these two waves are light waves then the phenomenon is called ***interference of light.*** At the point of interference, the resultant intensity will be maximum or minimum. The most common examples of interference of light waves are change in colours of a soap bubble, colours exhibited by thin films floating on thick oils, colours of peacock tail, etc.

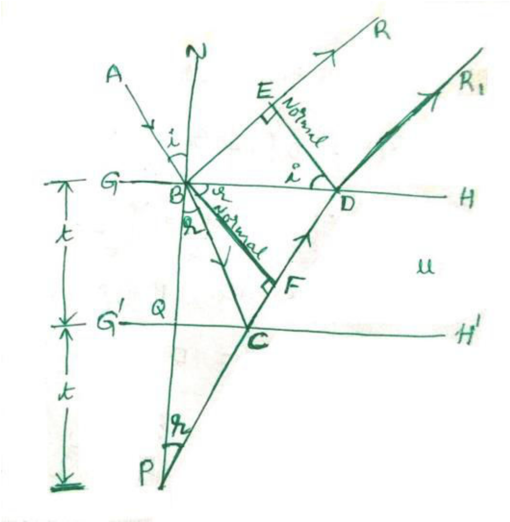
Interference in thin films:

The colours in thin films, soap bubbles & oil slicks are due to interference in thin films. The formation of interference is by division of amplitude.

If a plane wave is incident on a thin surface, then a wave reflected from the upper surface interferes with the wave reflected from the lower surface so that interference pattern is formed.

Interference in thin films by reflection [Cosine law in optics]:

Let GH and G^1H^1 are the surfaces of a transparent film of thickness t whose refractive index is μ.Consider a ray AB is incident on the upper surface of the film. It is reflected along BR and transmitted along BC. After reflection at C the ray takes the path CD and gets refracted at D and finally comes out along DR_1 in air. It is clear that BR is parallel to DR_1. Draw a normal DE on BR and normal BF on DC. Extend DC in backward direction which meets BQ at P. From the figure $\langle ABN = i$, the angle of incidence and $\langle QBC = r$, the angle of refraction. From the geometry $\langle BDE = i$ and $\langle DBF = \langle CPQ = r$



The optical path difference between reflected rays BR and DR_1 is given by,

$(BC + CD)$ in the medium - BE in air medium

$$\Delta = \mu(BC + CD) - BE$$

We know that from Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

From ΔEDB ,

$$\sin i = \frac{BE}{BD}$$

From ΔDBF ,

$$\sin r = \frac{FD}{BD}$$

$$\mu = \frac{\sin i}{\sin r} = \frac{BE/BD}{FD/BD} = \frac{BE}{FD}$$

$$BE = \mu(FD)$$

Therefore,

$$\Delta = \mu(BC + CD) - FD$$
$$= \mu(BC + CD - FD)$$

But from the figure we have,

$$CF + FD = CD$$
$$CF = CD - FD$$
$$\therefore \Delta = \mu(BC + CF)$$

But, $BC = CP$

$$\therefore \Delta = \mu(BC + CF)$$
$$= \mu(CP + CF)$$

But from the figure, $CP + CF = PF$

$$\therefore \Delta = \mu(PF)$$

From ΔBPF ,

$$\cos r = \frac{PF}{BP}$$

$$= \frac{PF}{2t}$$

$$2t \cos r = PF$$

$$\therefore \Delta = 2\mu t \cos r$$

An additional path difference of $\lambda/2$ is introduced as the first ray is reflected by denser medium. Therefore, the optical path difference is,

$$\therefore p = 2\mu t \cos r + \lambda/2$$

Condition for maximum:

The intensity of light is maximum when the constructive interference takes place that is path difference is $n\lambda$

$$2\mu t \cos r + \lambda/2 = n\lambda$$

$$2\mu t \cos r = \left(n - \frac{1}{2}\right)\lambda$$

$$= (2n\text{-}1)\frac{\lambda}{2}$$

This is the condition for bright fringe.

Condition for minimum:

The intensity of light is minimum when the destructive interference takes place that is

Optical path difference is $(2n+1)\frac{\lambda}{2}$

$$2\mu t \cos r + \lambda/2 = (2n+1)\frac{\lambda}{2}$$

$$= n\lambda$$

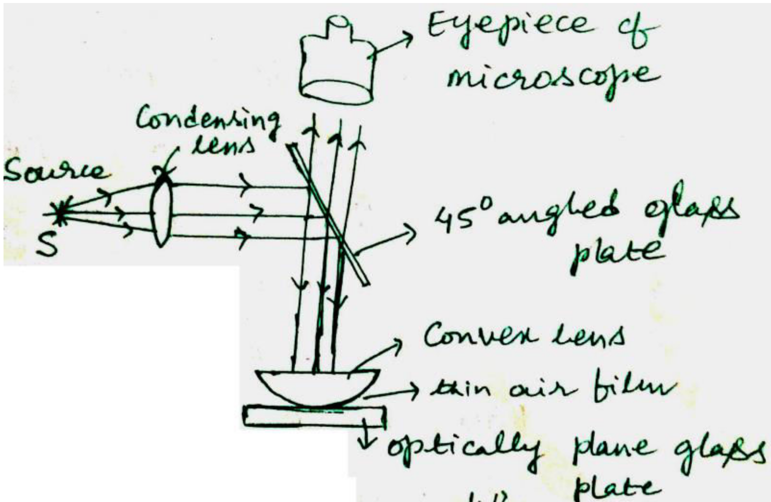
$$2\mu t \cos r = n\lambda$$

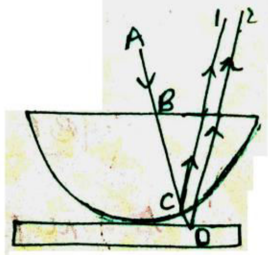
This is the condition for dark fringe.

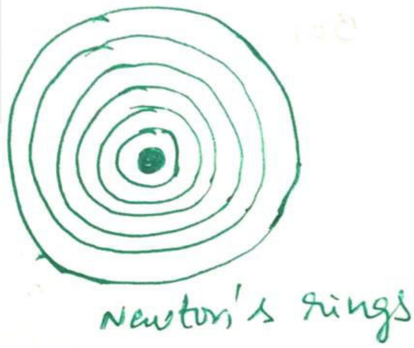
NEWTON'S RINGS:

The formation of Newton's rings is based on the principle of interference in thin films. A convex lens of large focal length **f** is placed on an optically plane glass plate such that a thin film of air is formed between these two as shown in the figure. When a Plano convex lens with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between these two. The thickness of the air film at the point of contact is zero. If monochromatic light is allowed to fall normally and the film is viewed in reflected light, alternate dark and bright rings concentric around the point of contact between the lens and glass plate are observed.

Experimental arrangement:







The experimental arrangement of Newton's rings is as shown in the figure. A Plano convex lens with its convex surface is placed on a plane glass plate P. The lens makes contact with the glass plate. Light from a monochromatic source such as sodium lamp falls on a glass plate held at an angle of 45^o with the vertical direction. The glass plate reflects normally a part of the incident light towards the air film enclosed by the lens and the glass plate. A part of the incident light is reflected by the curved surface of the lens and a part is transmitted which is reflected back from the plane surface of the plate. These two reflected light rays interfere and gives rise to an interference pattern in the form of circular rings. These rings are realized in the air film, and can be seen with a microscope focused on the film.

Explanation of the formation of Newton's rings:

Newton's rings are formed due to interference between the waves reflected from the top and bottom surface of the air film formed between the convex lens and glass plate. The formation of Newton's rings can be explained with the help of the figure shown above. AB is a monochromatic ray of light, which falls on the system. A part is reflected at C (glass air boundary) which goes out in the form of ray-1 without any phase reversal. The other part is refracted along CD. At point D it is again reflected and goes out in the form of ray-2 with a phase reversal of π. The reflected rays 1 and 2 are in a position to produce interference fringes as they have been derived from the same ray AB and hence fulfill the condition of interference.

The rings are observed in the reflected light, the path difference between them is

$$P = 2\mu t \cos r + \lambda / 2$$

For air film, $\mu = 1$ and for normal incidence,

$$\langle i = \langle r = 0$$

Therefore,

$$P = 2t + \lambda/2$$

At the point of contact $t = 0 \Rightarrow P = \lambda/2$ which is the condition for minimum intensity. Thus, the central spot is always dark.

The condition for maximum is $2t + \frac{\lambda}{2} = n\lambda \Rightarrow 2t = (2n-1)\frac{\lambda}{2}$

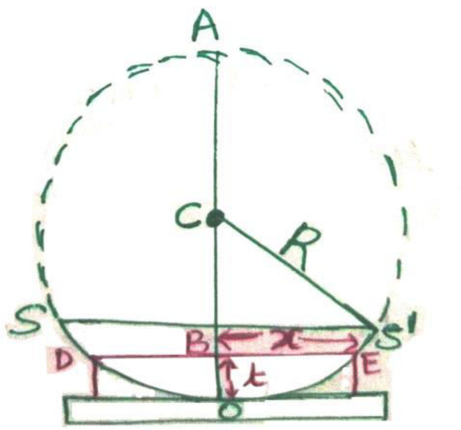
and the condition for minimum is $2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \Rightarrow 2t = n\lambda$ where n = 0,1,2,3 …

The above expressions shows that a maximum of a particular order 'n' will occur for a constant value of 't'. In case of this system 't' remains constant along a circle. Thus the maximum or minimum is in the form of a circle. For a different value of 't', different maxima and minima will occur.

Diameter of the bright and the dark rings:

Let $SOS^!$ be the lens placed on the glass plate $GG^!$

The curved surface $SOS^!$ is the part of the spherical surface which is shown in dotted lines in the below figure with centre C. Let R be the radius of the curvature and x be the radius of Newton's rings corresponding to constant film of thickness t.



The condition for n^{th} maxima is ,

$$2t + \lambda/2 = n\lambda$$

$$2t = \left(n - \frac{1}{2}\right)\lambda$$

$$2t = (2n-1)\frac{\lambda}{2} \,.......(1)$$

The condition for n^{th} minima is,

$$2t + \lambda/2 = \left(n + \frac{1}{2}\right)\lambda$$

$$2t = n\lambda \,.......(2)$$

From the figure,

$BD.BE = AB.BO \quad$ (from the law of chords)

$x.x = (2R - t)t$

$x^2 = 2Rt - t^2$

As t is small, hence t^2 (tends to zero) can be neglected,

$x^2 = 2Rt$

$2t = \frac{x^2}{R}$ ....(3)

From equations (1) and (3), for bright ring we have

$\frac{x^2}{R} = (2n-1)\frac{\lambda}{2}$

$2x^2 = R(2n-1)\lambda$

$(2x)^2 = 2R(2n-1)\lambda$

But $2x = D$, the diameter of the Newton's ring.

$D^2 = 2R(2n-1)\lambda$

$D = \sqrt{(2n-1)2R\lambda}$

$= \sqrt{(2n-1)}\sqrt{2R\lambda}$

$D_n \infty \sqrt{(2n-1)}$ ....(4)

Thus, the diameter of bright rings is proportional to the square root of natural odd numbers as $(2n-1)$ is odd.

Similarly, for dark bands we have from equations (2) and (3)

$\frac{x^2}{R} = n\lambda$

$x^2 = nR\lambda$

$(2x)^2 = 4nR\lambda$

$D_n^2 = 4nR\lambda$ ....(5)

$D_n = \sqrt{4nR\lambda}$

$D_n = \sqrt{n}\sqrt{4R\lambda}$

$D_n \infty \sqrt{n}$ ....(6)

Thus, the diameter of dark rings is proportional to the square root of all natural numbers. This is for reflected system.

Note: It can be seen that fringe width decreases with the order of the fringe and fringes get closer with increase in their order.

Determination of the Radius of curvature of the given Convex lens: -

If D_m and D_n the diameters of m^{th} and n^{th} dark rings then from equation (5),

$D_m^2 = 4mR\lambda$ and $D_n^2 = 4nR\lambda$

$$D_m^2 - D_n^2 = 4(m-n)R\lambda$$

$$R = \frac{D_m^2 - D_n^2}{4(m-n)\lambda} \quad ....(7)$$

Determination of the wavelength of the given unknown light source: -

From equation (7) we can determine the radius of curvature of given convex lens provided wavelength is known.

$$\lambda = \frac{D_m^2 - D_n^2}{4R(m-n)} ....(8)$$

From equation (7) we can determine the wavelength of given sodium light provided radius of curvature of convex lens is known.
Similarly, for bright ring also we can show that the above equations are equally applicable.

Determination of Refractive index of an unknown liquid:

The liquid whose refractive index is to be determined is placed between Plano-convex lens and glass plate. The diameters of m^{th} and n^{th} rings are determined. If D_m^1 and D_n^1 are the diameters of m^{th} and n^{th} rings respectively then,

$$\left(D_m^1\right)^2 - \left(D_n^1\right)^2 = \frac{4R(m-n)}{\mu}\lambda ....(9)$$

Here, R is the radius of the curvature of Plano-convex lens, λ is the wavelength of the monochromatic light and μ is the refractive index of the liquid.

When the space between convex lens and glass plate is air, then the diameters of the m^{th} and n^{th} rings are determined by,

$$D_m^2 - D_n^2 = 4(m-n)R\lambda ....(10)$$

Dividing equation (9) and (10) we get,

$$\mu = \frac{D_m^2 - D_n^2}{\left(D_m^1\right)^2 - \left(D_n^1\right)^2} ....(11)$$

From equation (11) refractive index of unknown liquid can be determined.

Numerical Problems with Solutions

1.Two coherent sources of intensity 16 Wm^{-2} and 25 Wm^{-2}interfere to form fringes. Find the ratio of maximum intensity to minimum intensity?

Sol: ***Given data:*** I_1=16 Wm^{-2} & I_2=25 Wm^{-2} ; I_{max} : I_{min} = ?

Intensity, $I = a^2$ is the square of amplitude

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$

So,

$$\frac{a_1^2}{a_2^2} = \frac{16}{25} \Rightarrow \frac{a_1}{a_2} = \frac{4}{5} \Rightarrow a_1 = \left(\frac{4}{5}\right)a_2 \Rightarrow a_2 = 1.25a_1$$

The ratio of I_{max} and I_{min} is,

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(a_1 + 1.25a_1)^2}{(a_1 - 1.25a_1)^2} = 81$$

The ratio of maximum intensity to minimum intensity = 81:1

2.A soap film of refractive index 1.33 and thickness 5000A° is exposed to white light. What wavelengths in the visible region are reflected?

Sol: ***Given data:*** Refractive index of soap $\mu = 1.33$

Thicknesses of soap film $t = 5000\ \text{A}^\circ = 5000 \times 10^{-8}$ cm

Wavelength $\lambda_{visible}$ =?

The incident light reflected on both surfaces of film combine to produce interference. So, the condition for constructive interference is used for reflection.

The condition for maximum reflection in thin film is,

$$2\mu t\cos r = (2n+1)\lambda/2 \qquad \text{for } (n = 0,1,2,3....)$$

Here, $r = 0^\circ$ (for maximum reflection)

$$\lambda = \frac{4\mu t\cos r}{(2n+1)}$$

$$= \frac{4(1.33)(5000\times10^{10})\cos 0^\circ}{(2n+1)}$$

$$= \frac{26600}{(2n+1)}$$

For

$n = 0 \quad \lambda_1 = 266000\ \text{A}^\circ$

$n = 1 \quad \lambda_1 = 8867\text{A}^\circ$

$n = 2 \quad \lambda_1 = 5320\text{A}^\circ$

$n = 3 \quad \lambda_1 = 3800\text{A}^\circ$

The wavelength in the visible region is 5320A°

3.In a Newton's rings system if the 4th and 6th dark rings are found to be 3 mm and 3.6 mm, calculate the wavelength of light used. The radius of curvature of the convex surface of the lens is 0.9 m.

Sol: ***Given data:*** The diameter of 4th ring, $D_n = D_4 = 3\ \text{mm} = 3\times10^{-3}$ m

The diameter of 6th ring, $D_m = D_6 = 3.6\ \text{mm} = 3.6\times10^{-3}$ m

The radius of curvature of convex lens is $R = 0.9\ \text{m}$; λ=?

The expression for wavelength of light is, $\lambda = \dfrac{D_m^2 - D_n^2}{4(m-n)R}$

Here, $m = 6 \quad n = 4$, $\lambda = \dfrac{D_6^2 - D_4^2}{4(6-4)0.9} = \dfrac{(3.6\times10^{-3})^2 - (3\times10^{-3})^2}{4\times2\times0.9} = 5500\ \text{A}^\circ$

The wavelength of light = $5500\ \text{A}^\circ$

4. In a Newton's rings experiment the diameters of 5th and 15th ring is 0.336 cm and 0.59 cm respectively. If the radius of curvature of the Plano convex lens is 100 cm. find the wavelength of monochromatic light. What happens to the ring diameters when the air film is replaced with coconut water of refractive index 1.33?

Sol: ***Given data:*** The diameter of 5th ring, $D_n = D_5 = 0.336$ cm

The diameter of 15th ring, $D_m = D_{15} = 0.59$ cm

The radius of curvature of convex lens is $R = 100\ \text{cm}$; λ=?

The expression for wavelength of light is, $\lambda = \dfrac{D_m^2 - D_n^2}{4(m-n)R}$

Here, $m = 15 \quad n = 5$, $\lambda = \dfrac{D_{15}^2 - D_5^2}{4(15-5)100} = \dfrac{(0.59)^2 - (0.336)^2}{4 \times 10 \times 100} = 5880\ \text{A}^\circ$

The ring diameters are,

$$\mu = \frac{D_m^2 - D_n^2}{D_m^{!2} - D_n^{!2}}$$

$$D_m^{!2} - D_n^{!2} = \frac{(0.59)^2 - (0.336)^2}{1.33}$$

$$= 0.1768\ \text{cm}$$

Therefore ring diameters are decreases when the air film is replaced with coconut water film.

5.A parallel beam of sodium light of wavelength 5893A° is incident at an angle of 45° on a film of mustered oil (μ=2.6) on water. Calculate the smallest thickness of the film which will make it appear dark?

Sol: ***Given data:*** Wavelength of sodium light, $\lambda = 5893\ \text{A}^\circ = 5893 \times 10^{-8}$ cm

Refractive index of mustered oil, $\mu = 2.6$

Angle of incidence, $\angle \text{i} = 45^\circ$; t=?

The condition for dark band in thin film is,

$$2\mu t \cos r = n\lambda$$

Here, μ is the refractive index, t is thickness, r is angle of refraction and λ is wavelength.

To determine the angle of refraction,

$$\mu = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{\mu} \Rightarrow \sin r = \frac{\sin 45^\circ}{2.6} = 0.327$$

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (0.327)^2} = 0.9449$$

The thickness of the film is,

$$2\mu t \cos r = n\lambda \Rightarrow t = \frac{n\lambda}{2\mu \cos r} = \frac{(1)(5893 \times 10^{-8})}{2(2.6)(0.9449)} = 1.1 \times 10^{-5}\ \text{cm}$$

Thickness of the film t= 1.1×10^{-5} cm

6. In Newton's rings experiment the diameter of the 10th ring changes from 1.40 cm to 1.27 cm when a liquid is introduced between the lens and the plate? Calculate the refractive index of liquid?

Sol: Given data:

The diameter of dark ring is, $D_n = 1.40$ cm

The diameter of dark ring in liquid surface is, $D_n^1 = 1.27$ cm

The refractive index of liquid is, $\mu = \dfrac{D_n^2}{D_n^{(1)^2}} = \dfrac{(1.40)^2}{(1.27)^2} = 1.21$

1.2. DIFFRACTION

When the light falls on an obstacles or small apertures whose size (d) is in comparable with the wavelength of light (λ), there is a departure from straight line propagation. The light bends round the corners of obstacles or apertures and enters in the region of geometrical shadow. This bending of light is called ***diffraction***. The amount of bending depends upon the size of the obstacle and the wavelength of the light. The phenomenon of diffraction produces bright and dark fringes known as diffraction bands or diffraction fringes.

Condition for diffraction

Wavelength of wave (λ)	Size of the obstacle or aperture (d)	Occurrence of diffraction
λ <<d		×××
λ >> d		×××
$\lambda \sim \cong d$		√√√

Diffraction patterns can be observed with any kind of waves like sound, light & water waves. Ocean waves can diffract around jetties and other obstacles. Sound waves bends at the corners of doors and windows, so that one can able to hear the sound generated in the neighboring room. Condition of diffraction is **$\lambda \sim d$.**

DIFFERENCES BETWEEN FRESNEL'S AND FRAUNHOFFER DIFFRACTIONS:

The diffraction phenomenon is divided into two classes.

(i) Fresnel's diffraction
(ii) Fraunhoffer diffraction

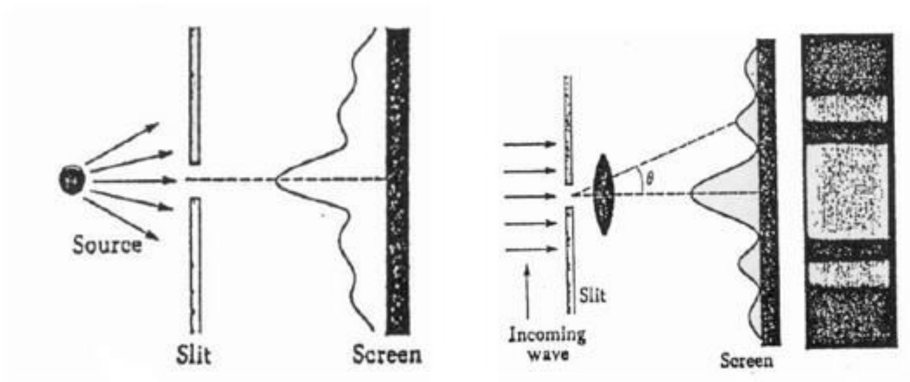


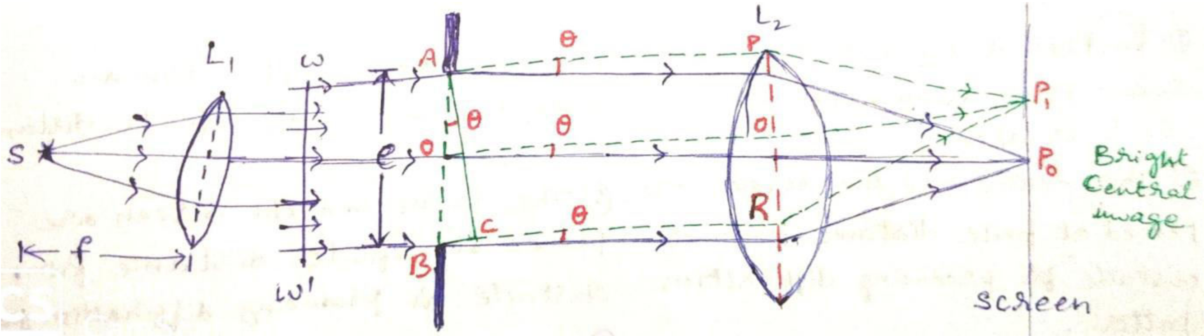
Fig.1 Fresnel diffraction and Fraunhoffer diffraction

Fresnel's diffraction	Fraunhoffer diffraction
(1) In Fresnel diffraction a point source or an illuminated narrow slit is used.	(1) In Fraunhoffer diffraction an extended source at infinite distance is used.
(2) The source and screen are placed at a finite distance from the obstacle for producing diffraction.	(2) The source and screen are placed at infinite distance from the obstacle for producing diffraction pattern.
(3) No lens is used to focus the light rays.	(3) A convex (converging) lens is used to focus the parallel rays.
(4)The wave front undergoing diffraction is either spherical or cylindrical.	(4) The wave front undergoing diffraction is plane wave front.
(5)Distances are important in this class of	(5) Angles (Angular inclinations) are

diffraction.	important in this class of diffraction.
(6) The light rays proceed directly to axial points.	6) There is a large no. of parallel rays falling on the lens corresponding to each other.
(7) The centre of diffraction pattern may be bright or dark	(7) The centre of diffraction pattern is always bright
(8)Mathematical investigations are complicated and only approximate	(8)Mathematical investigations are rigorous and easy.
(9)Theoretical explanation is not simple	(9)Theoretical explanation is simple
(10)Fresnel diffraction is near field diffraction.	(10)Fraunhoffer diffraction is far field diffraction.

FRAUNHOFFER DIFFRACTION DUE TO SINGLE SLIT:

When a plane wave front $ww^{|}$ of monochromatic wavelength λ incident normally on the slit, AB is a narrow slit of the width e diffracted light can be focused by the convex lens on the screen. Every point of the wave front in the plane of the slit is a source of secondary wavelets which spread out in all directions. The secondary wavelets which travel normal to the slit are focused at P_0 by the lens. Hence, P_0 is bright central image. The point of minimum or maximum intensity depending upon the path difference between the secondary waves



Theory:

Draw a perpendicular AC on BR, the path difference between the rays is BC. As per the figure,

$$\sin\theta = \frac{BC}{AB}$$

$$BC = AB\sin\theta$$

$$= e\sin\theta$$

Path difference, $BC = e\sin\theta$

Phase difference = $\frac{2\pi}{\lambda}(e\sin\theta)$

Let the width of the silt is divided into n number of elements and the amplitude of each wave from each part is a.

The phase difference between any two consecutive waves is,

$$d = \frac{\text{total phase difference}}{\text{total number of elements}}$$

$$d = \frac{2\pi e\sin\theta}{\lambda n}$$

By the vector addition using polygon method, the amplitudes of successive waves are added and the resultant amplitude is given by,

$$R = \frac{a \sin nd/2}{\sin d/2}$$

$$= \frac{a \sin \mathrm{n}\left(\frac{2\pi}{n\lambda}\left(\frac{e \sin \theta}{2}\right)\right)}{\sin\left(\frac{2\pi}{n\lambda}\left(\frac{e \sin \theta}{2}\right)\right)}$$

$$= \frac{a \sin\left(\frac{\pi}{\lambda}(e \sin \theta)\right)}{\sin\left(\frac{\pi}{n\lambda}(e \sin \theta)\right)}$$

Let, $\alpha = \frac{\pi e \sin \theta}{\lambda}$

$$R = \frac{a \sin \alpha}{\sin\left(\frac{\alpha}{n}\right)} \qquad \text{for } \frac{\alpha}{n} \text{ is small, } \sin\left(\frac{\alpha}{n}\right) \approx \frac{\alpha}{n}$$

$$= \frac{an \sin \alpha}{\alpha} \qquad \text{Here, } A = an$$

$$= \frac{A \sin \alpha}{\alpha}$$

But the intensity is directly proportional to square of amplitude.

$$I = \frac{A^2 \sin^2 \alpha}{\alpha^2}$$

$$= I_0 \frac{\sin^2 \alpha}{\alpha^2} \qquad \text{where, } I_0 = A^2$$

Intensity distribution:

Intensity of Central maximum: Central maximum occurs when $\theta = 0^\circ$ and hence α is zero.

$$R = \frac{A \sin \alpha}{\alpha}$$

The value of $\frac{\sin \alpha}{\alpha}$ → to unity

$$= \frac{A}{\alpha}\left(\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!+} + \ldots\ldots\right)$$

$$= \frac{A}{\alpha}(\alpha)\left(1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!+} + \ldots\ldots\right)$$

If $\alpha = 0$, then $R = A$ that is $I = I_0$ the intensity is maximum.

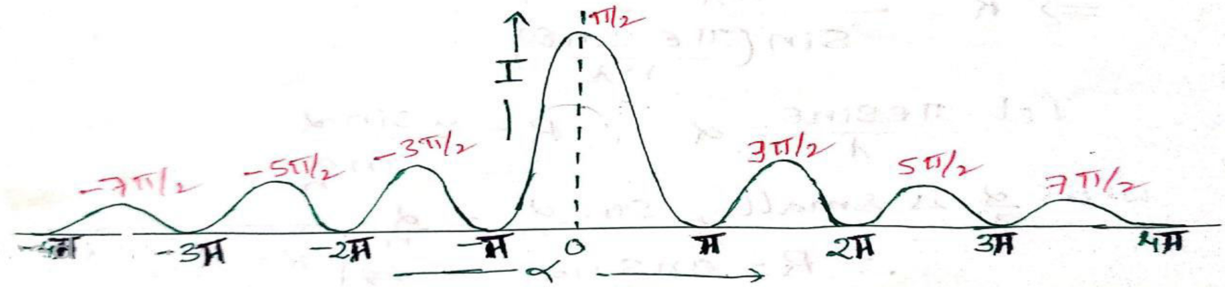
Intensity of Secondary maxima: In addition to principal maximum at $\theta = 0$, there are weak secondary maxima between equally spaced minima, when $\alpha = \pm(2n+1)\frac{\pi}{2}$, where $n = 1,2,3....$

($n = 0 \rightarrow$ to central maximum). If

$$n = 1, I_{\max} = \pm 3\frac{\pi}{2}$$

$$n = 2, I_{\max} = \pm 5\frac{\pi}{2}$$

$$n = 3, I_{\max} = 7\frac{\pi}{2}$$



Intensity of Secondary minima:

Intensity is minimum when $\alpha = \pm n\pi$, $n = 1,2,3,....$

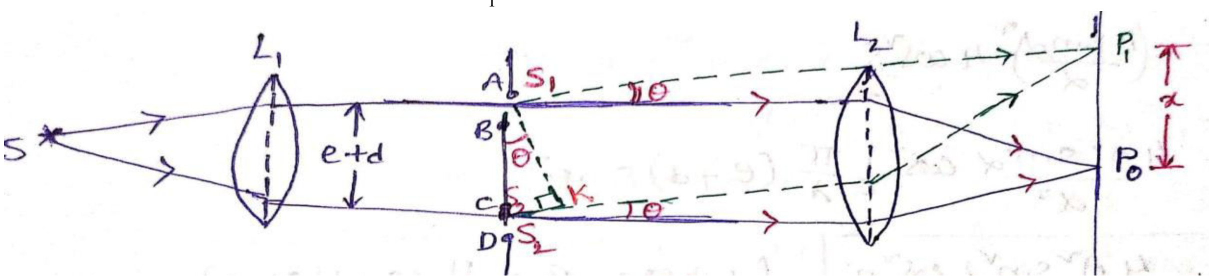
$$I = I_0\left(\frac{\sin^2\alpha}{\alpha}\right) = 0$$

As shown in the figure, diffraction pattern consists of a central maximum occurs in the direction of incident ray as secondary maxima of decreasing intensity on either side when $\alpha = \pm\pi/2, \pm 3\pi/2$

Between the secondary maxima there are secondary minima when $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$ etc

FRAUNHOFFER DIFFRACTION DUE TO DOUBLE SLIT:

Let AB and CD be two parallel slits of equal with e and separated by opaque distance d. The distance between mid-points of slits is $e+d$. The diffraction of the two slits is combination of diffraction as well as interference. When the plane wave front is incident on both the slits all the points within the slit becomes the source of secondary wavelets which travel in all direction. The secondary wavelets travelling in the direction of incident light come to focus at P_0 while the secondary wavelets travelling in the direction making an angle θ with the incident direction comes to focus at P_1



According to the diffraction at single slit, the resultant amplitude R is given by,

$$R = \frac{A\sin\alpha}{\alpha} \quad \text{where } \alpha = \frac{\pi e \sin\theta}{\lambda}$$

The path difference between S_1 and S_2 in the direction $\theta = S_2K = (e+d)\sin\theta$

The phase difference is,

$$\delta = \frac{2\pi}{\lambda}(\text{path difference})$$

$$= \frac{2\pi}{\lambda}(e+d)\sin\theta$$

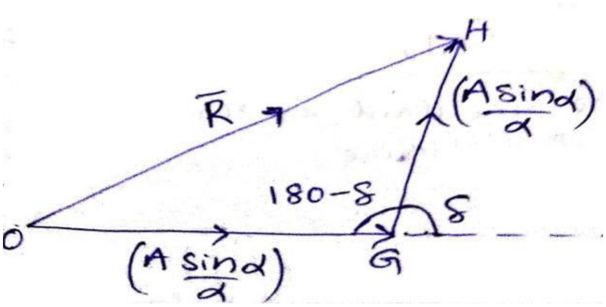
The resultant amplitude R at a point P_0 is given with the help of following figure.

From the figure,

$$OH^2 = OG^2 + GH^2 + 2(GH)(OH)\cos\delta$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{A\sin\alpha}{\alpha}\right)\cos\delta$$

$$= \left(\frac{A\sin\alpha}{\alpha}\right)^2 (1+1+2(1)(1)\cos\delta)$$



$$R^2 = 2\left(\frac{A\sin\alpha}{\alpha}\right)^2 (1+\cos\delta)$$

$$= \left(\frac{A\sin\alpha}{\alpha}\right)^2 2\left(1+2\cos^2\frac{\delta}{2}-1\right)$$

$$= \left(\frac{A\sin\alpha}{\alpha}\right)^2 4\cos^2\frac{\delta}{2}$$

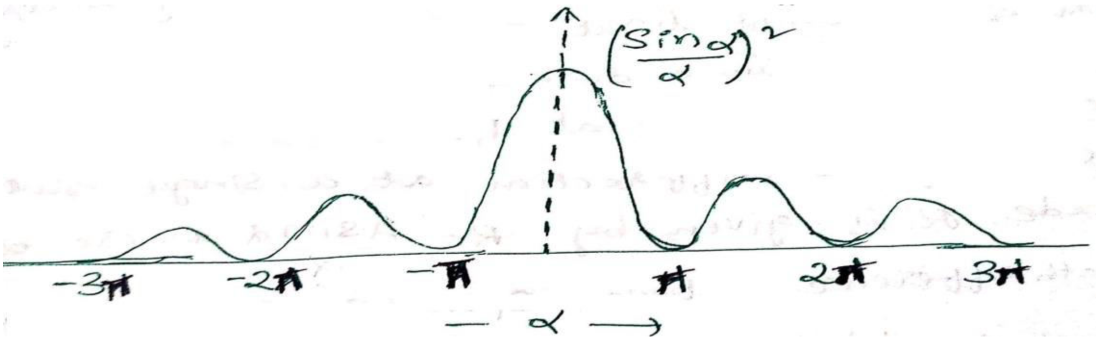
$$R^2 = 4\frac{A^2\sin^2\alpha}{\alpha^2}\cos^2\beta \qquad \left(\text{let } \beta = \frac{\delta}{2}\right)$$

$$R^2 = 4\frac{A^2\sin^2\alpha}{\alpha^2}\cos^2\left(\frac{\pi(e+d)\sin\theta}{\lambda}\right) \quad \text{where, } \beta = \frac{\pi(e+d)\sin\theta}{\lambda}$$

The resultant intensity at P_0 is,

$$I = R^2 = 4\frac{A^2\sin^2\alpha}{\alpha^2}\cos^2\beta$$

The resultant intensity depends on $\frac{A^2\sin^2\alpha}{\alpha^2}$ and $\cos^2\beta$. The diffraction term $\frac{\sin^2\alpha}{\alpha^2}$ gives the central maximum in the direction $\theta = 0^\circ$ and having alternate secondary maximum of decreasing intensity on either side as shown in the figure.



Secondary minima or minimum intensity:

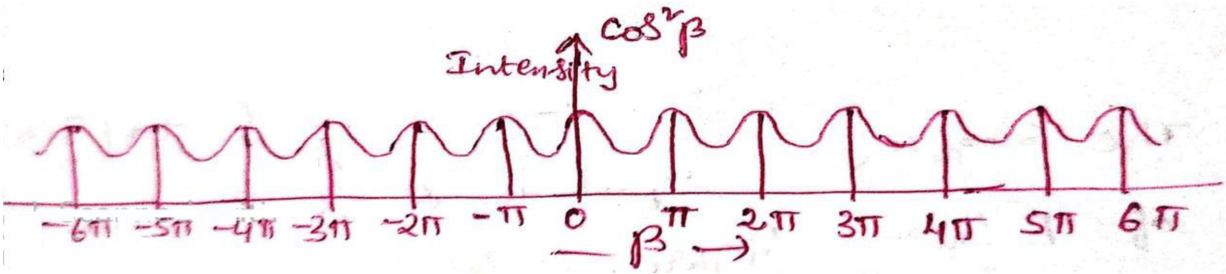
The minimum intensity occurs when, $\sin\alpha = 0$ but $\alpha \neq 0$

$$\Rightarrow \alpha = \pm m\pi$$

$$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

$$e\sin\theta = \pm m\lambda$$

The interference from $\cos^2\beta$ gives a set of equidistant dark and bright fringes as shown below



Secondary maxima:

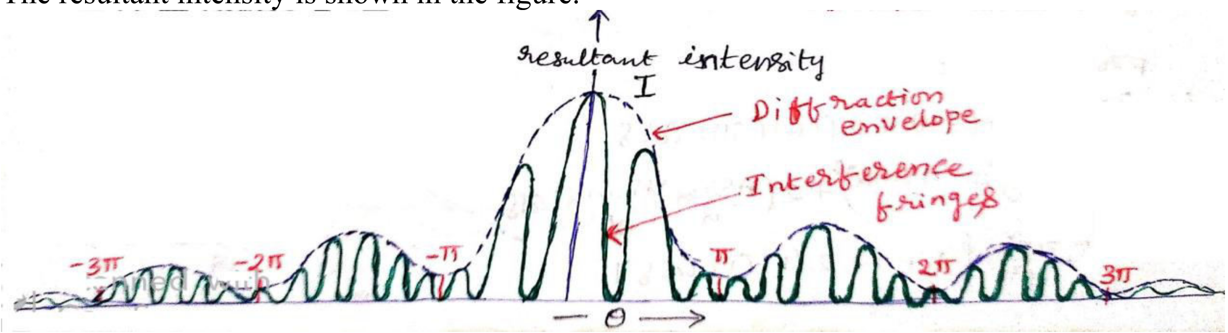
Secondary maxima occurs when $\cos^2\beta = 1$

$$\Rightarrow \beta = \pm n\pi$$

$$\frac{\pi(e+d)\sin\theta}{\lambda} = \pm n\pi$$

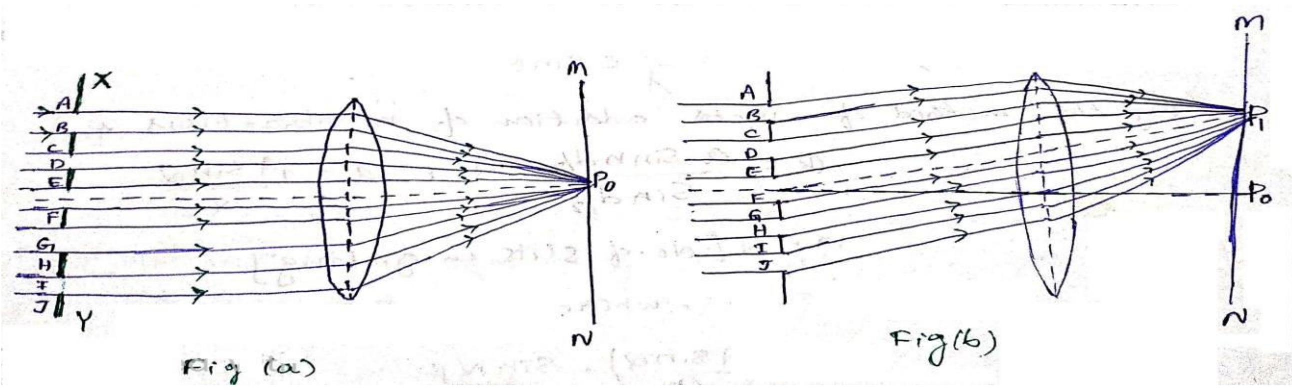
$$(e+d)\sin\theta = \pm n\lambda$$

The resultant intensity is shown in the figure.



FRAUNHOFFER DIFFRACTION DUE TO N-PARALLEL SLITS:

Consider $ABCDEFGIJ$ represent the section of grating normal to the plane of paper having width of each slit is uniform and is assumed as e. The slits AB, CD, EF, GH, IJ are separated by a distance d. The sum of two widths $e+d$ is called grating element or grating constant. It is also the distance between any two successive slits. Any two points on successive slits is called corresponding points. Let the plane waves of monochromatic light of wavelength λ incident normally on grating. By Huygens's principle each of the infinite points in the slit sends secondary wavelets in all directions. The secondary wavelets travelling in the same direction of incident light will come to focus at a point P_0 on the screen MN which is kept at the focal point of the convex lens as shown in figure. The secondary waves travelling in a direction inclined at an angle θ with normal rays are focused at P_1 on the screen which is different phases. As a result, dark and bright bands on both sides of central maxima are obtained.



Theory:

The path difference between any two consecutive rays is $(e+d)\sin\theta$.

The phase difference between any two consecutive rays is $\frac{2\pi}{\lambda}(e+d)\sin\theta$.

The wavelets from all points in a slit with direction θ are equivalent to a single wave of amplitude $\frac{A\sin\alpha}{\alpha}$ starting from the mid-point, where $\alpha = \frac{\pi}{\lambda}(e+d)\sin\theta$

By the method of vector addition of n vibrations,

$$R = \frac{a\sin nd/2}{\sin d/2} \quad \text{but } a = \frac{A\sin\alpha}{\alpha}$$

$$n = N(\text{number of slits on grating})$$

$$d = 2\beta, \quad \text{where } \beta = \frac{\pi}{\lambda}(e+d)\sin\theta$$

$$\therefore R = \left(\frac{A\sin\alpha}{\alpha}\right)\frac{\sin N\beta}{\sin\beta}$$

$$\text{Intensity } = R^2$$

$$I = A^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)\frac{\sin^2 N\beta}{\sin^2\beta}$$

The first term $\frac{\sin^2\alpha}{\alpha^2}$ gives the intensity due to single slit. Second term $\frac{\sin^2 N\beta}{\sin^2\beta}$ gives the intensity due to N slits.

Intensity distribution:

Intensity of Principal Maxima:

The intensity will be maximum when $\sin\theta = 0$ and $\sin\beta = 0$

The maxima are obtained for,

$$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

$$(e+d)\sin\theta = \pm n\lambda, \quad n = 0,1,2,3.....$$

Intensity of Minima:

The condition for minima occurs when

$$\sin N\beta = 0 \text{ but } \sin\beta \neq 0$$

$$N\beta = \pm m\pi$$

$$N\left(\frac{\pi}{\lambda}\right)(e+d)\sin\theta = \pm m\pi$$

$$\Rightarrow N(e+d)\sin\theta = \pm m\lambda \quad \text{where} \quad m = 1,2,3.....$$

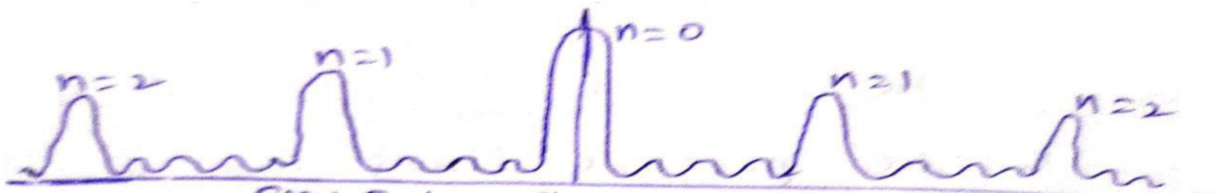


Fig: Intensity pattern of diffraction at N-slits.

DIFFRACTION GRATING:

An arrangement consists of large number of equidistant parallel slits of same width and separated by equal opaque spaces on a plane glass plate is called as diffraction grating. The corresponding diffraction pattern is known as grating spectrum.

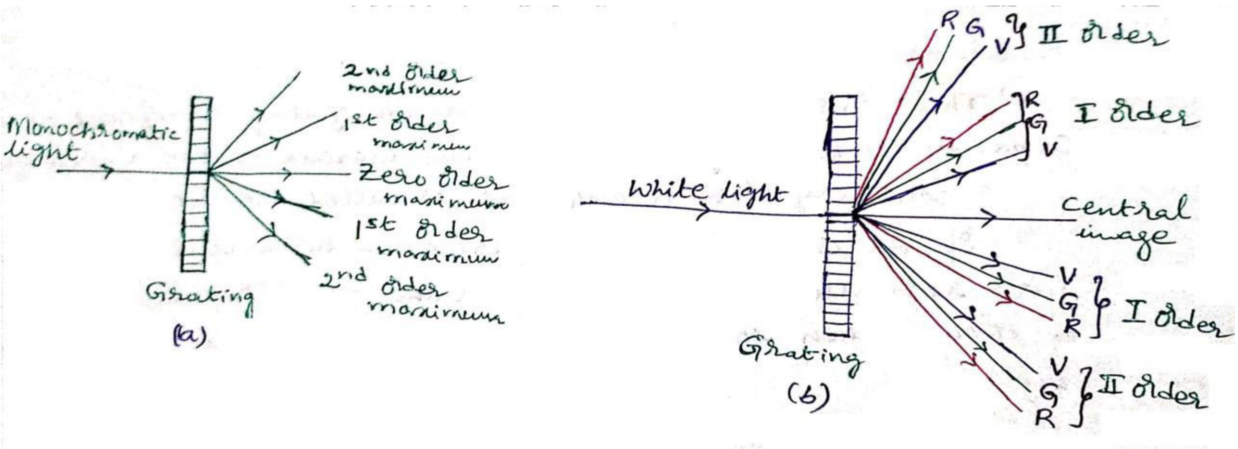
Gratings are made by ruling equidistant parallel lines on a plane glass plate by a fine diamond point. The ruled lines are opaque to light while space between the lines is transparent and acts as slit. This is known as plane transmission grating. A good quality grating consists of large number of slits about 15,000 lines per inch.

GRATING SPECTRUM:

The positions of the principal maxima are given by the equation

$$(e+d)\sin\theta = n\lambda \quad \text{where} \quad n = 0,1,2,3....$$

This relation is called grating equation. Here, $(e+d)$ is called grating element, n is the order of the spectrum and λ is the wavelength of incident light. The angle of diffraction depends upon the wavelength. The corresponding spectrum is called grating spectrum.



The first order spectrum is obtained for $n=1$ then $(e+d)\sin\theta 1=\lambda$

The second order spectrum is obtained for $n=2$ then $(e+d)\sin\theta 2=2\lambda$ and so on…

Thus, different order bright images are obtained on both sides of the principal maximum.

If white light is used then central maximum is white and for $n\neq 0$ in each order different colours are diffracted at different angles as shown in figure. Thus, by measuring the angles of diffraction for various colours one can determine the wavelengths.

Characteristics of grating spectrum:

(1) The spectra of different orders are situated symmetrically on both sides of central maximum.
(2) The spectral lines are straight, well defined and sharp.
(3) The spectral lines are in the order from violet to red.
(4) The spectral lines are more ordered and dispersed by increase in order.
(5) The most of the intensity goes to the central maximum and remaining is distributed for various orders.

DETERMINATION OF WAVELENGTH OF LIGHT BY DIFFRACTION GRATING:

The positions of the principal maxima in grating are given by equation,

‘ $(e+d)\sin\theta=n\lambda$ where $n=0,1,2,3....$

Here, $(e+d)$ is called grating element, n is the order of the spectrum and λ is the wavelength of incident light.

$$(e+d)\sin\theta=n\lambda$$

$$\lambda=\frac{(e+d)\sin\theta}{n}$$

$$\lambda=\frac{\sin\theta}{nN}\quad\left(\because\frac{1}{e+d}=N\right)$$

By knowing the values of angle of diffraction, order of the spectrum, the wavelength of light can be determined.

MAXIMUM NUMBER OF ORDERS POSSIBLE WITH A GRATING

There is a limit for any grating to observe the number of orders of diffraction spectrum. The grating equation is given by

$$(e+d)\sin\theta = n\lambda$$

$$\lambda = \frac{(e+d)\sin\theta}{n}$$

$$\lambda = \frac{\sin\theta}{nN} \quad \left(\because \frac{1}{e+d} = N\right)$$

$$n = \frac{\sin\theta}{N\lambda}$$

For the highest order in the grating θ =90^0, hence Sin 90^0=1

To observe the maximum number of possible orders of spectrum, the condition is Sinθ ≤ 1, Therefore, $n_{max}N\lambda \le 1$

$$n_{max} \le \frac{1}{N\lambda}$$

This is the condition for maximum number of possible orders from a grating

Resolving power:

The ability of an optical instrument to form distinctly separate images of two objects which are very close together is called its resolving power.

RAYLEIGH CRITERIA FOR RESOLUTION:

"Two spectral lines are resolvable by an optical instrument when the central maximum in the diffraction pattern of one form over the first minimum of the other and vice versa" This is known as Rayleigh criterion of Resolution.

RESOLVING POWER OF GRATING:

The resolving power of diffraction grating is defined as the capacity of to form separate diffraction maxima of two wavelengths which are very close to each other.

If $d\lambda$ is the smallest difference in the two wavelengths which are just resolvable by grating and λ is either or mean wavelength of them, then the resolving power of grating is measured by $\lambda / d\lambda$.

Let θ be the diffraction angle for λ.

And $\theta + d\theta$ be the diffraction angle for $\lambda + d\lambda$.

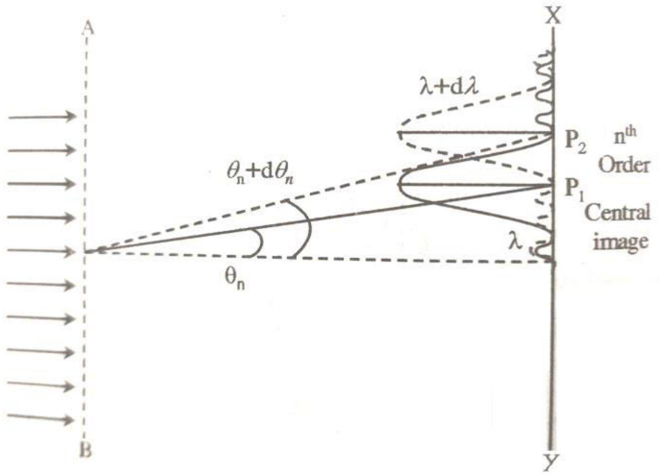
Principal maxima for λ is $(e+d)\sin\theta = n\lambda$ ......(1)

Principal maxima for $\lambda + d\lambda$ is $(e+d)\sin(\theta + d\theta) = n(\lambda + d\lambda)$ ....(2)

Minima for λ is $N(e+d)\sin(\theta + d\theta) = m\lambda$ ....(3)

Substitute $m = (Nn+1)$ in equation (3) then it becomes, (∵ According to Rayleigh criterion)

$$N(e+d)\sin(\theta + d\theta) = (Nn+1)\lambda \quad ....(4)$$



Multiply equation (2) with *N*,

$$N(e+d)\sin(\theta+d\theta)=Nn(\lambda+d\lambda) \quad ....(5)$$

From equations (4) and (5),

$$(Nn+1)\lambda = Nn(\lambda+d\lambda)$$
$$Nn\lambda+\lambda = Nn\lambda + Nnd\lambda$$
$$\lambda = Nnd\lambda$$
$$\frac{\lambda}{d\lambda} = Nn \quad ......(6)$$

$\frac{\lambda}{d\lambda}$ is directly proportional to the order of the spectrum & No. of lines on the grating

From equation (1),

$$n = \frac{(e+d)\sin\theta}{\lambda} \quad .....(7)$$

From (6) and (7),

$$\frac{\lambda}{d\lambda} = \frac{N(e+d)\sin\theta}{\lambda} \quad ......(8)$$

Equation (8) gives the measure of resolving power of grating

Dispersive Power of Grating:

The dispersive power of grating is defined as the rate of variation of the angle of diffraction θ with the wavelength λ.

The dispersive power of grating is,

$$D = \frac{d\theta}{d\lambda}$$

The condition for n^{th} order principle maximum is,

$$(e+d)\theta = n\lambda$$

Differentiating the above equation with respect to λ and θ

$$\frac{d\theta}{d\lambda}=\frac{n}{(e+d)\cos\theta}=\frac{nN}{\cos\theta}\left(\because \frac{1}{(e+d)}=N\right)$$

The dispersive power of the grating is (i) directly proportional to order n of the spectrum (ii) inversely proportional to the grating element $(e+d)$ (iii) inversely proportional to $\cos\theta$

Model Problems on UNIT-1.2

1.A plane diffraction grating with 12000 lines/cm is used in second order with a light of wavelength 5000 A^0. Calculate the smallest wavelength difference that can resolve?

Sol: ***Given data:***

No. of lines on grating, $N = 12000$ lines/cm

Wavelength, $\lambda = 5000\ \text{A}^\circ = 5000\times10^{-8}$ cm

Order of the spectrum, $n = 2$

To get the smallest wavelength difference, the resolving power of grating is,

$$\frac{\lambda}{d\lambda}=nN \Rightarrow d\lambda=\frac{\lambda}{nN}=\frac{5000\times10^{-8}}{(2)(12000)}$$

$$d\lambda = 0.208\times10^{-8}\ \text{cm}$$

2 .A parallel beam of sodium light is normally incident on a plane transmission grating having 5000 lines per cm and a second order spectral line is observed at an angle of 30^0. Calculate the wave length of light.

Sol: ***Given data:***

No. of lines on grating is, $N = 5000$ lines/cm

Order of the spectrum, $n = 2$

Angle of diffraction, $\theta = 30^\circ$; The wavelength of light is,

$$\lambda=\frac{\sin\theta}{nN}=\frac{\sin 30^\circ}{(2(5000))}=5000\ \text{A}^\circ$$

3 .A grating has 6000 lines/cm. Find the angular separation between two wavelengths 500 nm and 510 nm in the third spectrum.

Sol: ***Given data:***

The two wavelengths, $\lambda_1 = 500\ \text{nm} = 5000\times10^{-8}$ cm and $\lambda_2 = 510\ \text{nm} = 5100\times10^{-8}$ cm

Number of lines on grating, $N = 6000$ lines/cm

The angular separation for λ_1 is $\sin\theta_1 = 3\times6000\times5000\times10^{-8} = 0.9$

$\theta_1 = 64^0 16^1$

The angular separation for λ_2 is $\sin\theta_2 = 3\times6000\times5100\times10^{-8} = 0.918$

$\theta_2 = 66^0 59^1$

Therefore the angular separation between two wavelengths$=66^0 59^1 - 64^0 16^1 = 2^0 48^1$

4 .Find the highest order that can be seen with a grating having 15000 lines per inch. The wavelength used is 600 nm.

Sol; ***Given data:***

No. of lines on grating, $N = 15000$ lines/inch $= 15000/2.54$ lines/ cm

Wavelength, $\lambda = 600\ \text{nm} = 6000\times10^{-8}$ cm

For highest order, $\theta = 90^\circ$; Maximum number of orders,

$$\lambda=\frac{\sin\theta}{nN}\Rightarrow n=\frac{\sin\theta}{\lambda N}=\frac{\sin 90^\circ}{(6000\times10^{-8}\ \text{cm})(5905.5)}=2.82$$

Therefore the highest order that can be seen = 2

1.3: POLARIZATION

The wave nature of light was established by the phenomenon of interference and diffraction. But these phenomena do not reveal the nature of light i.e., whether they are longitudinal or transverse. Light is made up of electric and magnetic fields which are perpendicular to each other and perpendicular to their direction of propagation. This transverse nature of light is explained by Polarization.

Light is symmetrical about its direction of propagation. Electric vector of light is resolved into two components, one is horizontal and other is vertical components. All the horizontal components are added and the vertical components are added. So, the beam of waves is represented by two waves of equal amplitudes vibrating in mutually perpendicular plane.

Representation of unpolarized light and plane polarized light:

The vibrations that are parallel to the plane of paper are π components represented by arrow marks and the vibrations that are perpendicular to the plane of paper are σ components represented by dots. In unpolarized light these components are combined as shown in the figure. Plane polarized light has vibrations that are confined to one particular direction. The property of achieving one sidedness is known as Polarization.

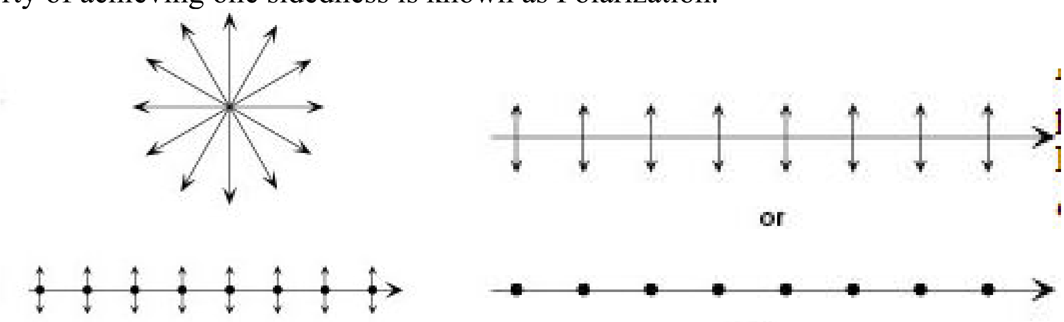


Fig: (a) Unpolarized light (b) Plane Polarized light

Polarization of waves:

The polarization of wave's concept is understood by taking the waves produced in stretched strings as analogy. The waves produced in stretched string are transverse in nature. Let AB be the string fixed at end B and held by hand at A as shown in the figure. By moving the hand up and down as well as sidewise, transverse vibrations are produced in the string and these vibrations move towards B. This is analogous to propagation of unpolarized light with vibrations perpendicular to the direction of motion.

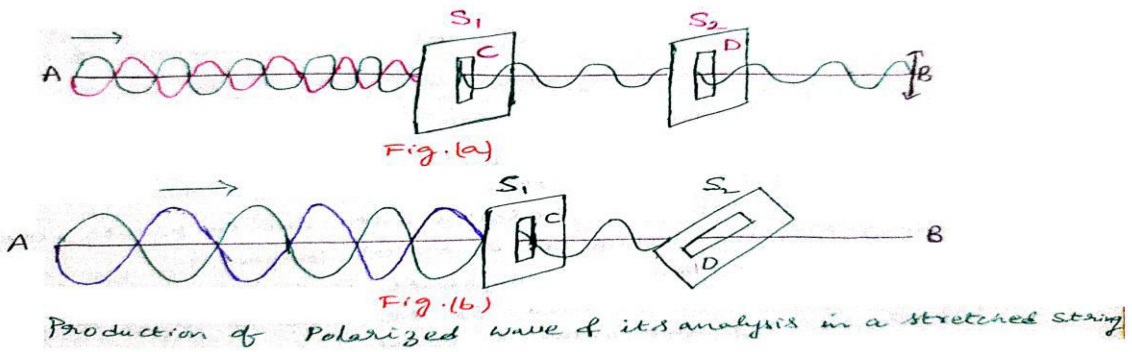
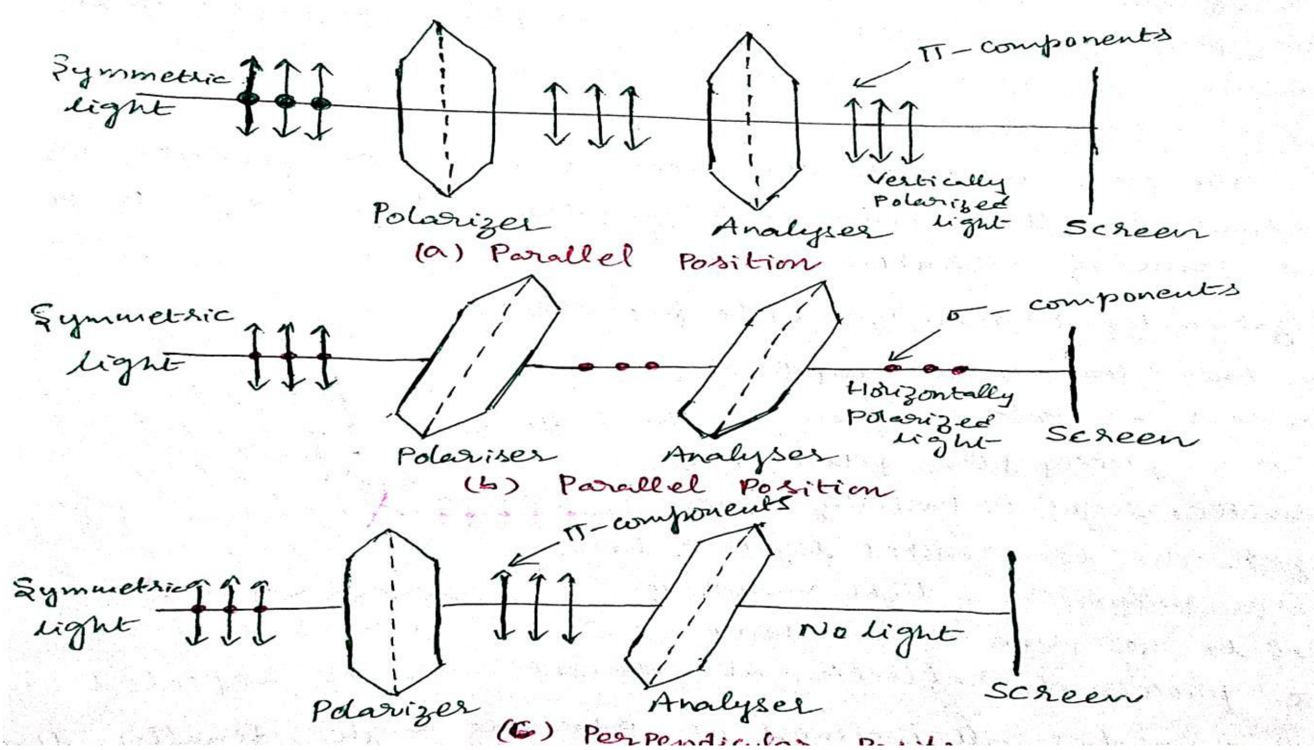


Fig.2: Production of polarized wave and its analysis in a stretched string

Suppose two slits S_1 and S_2 are the two slits between the points A and B. The string is passed through the slits; slit S_1 allows vibrations along the slit length only. Thus, the vibrations are confined to a single plane and hence the wave is said to be plane polarized.

Hence slit S_1 acts as polarizer. Let slit S_2 is held parallel to S_1. Then slit S_2 permits the vibrations to pass through it. If slit S_2 is rotated through 90°, no vibrations pass through S_2. Hence, S_2 acts as analyzer. When the same process is applied in case of unpolarized light, we obtain a plane polarized light. The production of polarized light is done by using crystals like tourmaline, quartz, calcite etc. when an ordinary light is passed through tourmaline crystal it allows vibrations parallel to its optic axis and cuts the vibrations perpendicular to it. So, light is not symmetric and confines to one plane. This is called plane polarized light.



This polarized light when allowed to fall on another tourmaline crystal the emergent light shows a variation in intensity. It is maximum when the two crystals are parallel and minimum when they are perpendicular. The first crystal is known as polarizer and the second crystal is analyzer.

Polarization: The light wave is said to be polarized if its electric vectors are not equal in all directions and the phenomenon of producing polarized light is known as polarization. (OR) The process of transforming unpolarized light into polarized light is known as **polarization.** (OR) The property of acquiring one-sidedness is called polarization.

Plane of polarization and plane of vibration:

When an ordinary light is passed through a tourmaline crystal, the light is said to be polarized and the vibrations are confined in one direction perpendicular to the direction of propagation of light.

Plane of vibration:

If the electric vector of the electromagnetic wave id confined to one plane such a light is known as plane polarized light and the plane is known as plane of vibration. From figure 4 CDEF represents plane of vibration.

Plane of polarization

The plane in which the electric vector components are zero or absent is called plane of polarization. It is always perpendicular to the plane of vibration. From figure 4 GHIJ represents the plane of polarization.

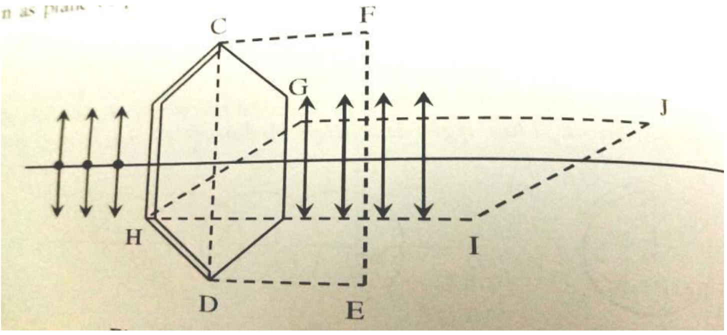


Fig 4: Representation of plane of vibration and polarization

Polarization of light can be achieved by transmission, reflection and double refraction.

Polarization by reflection or Brewster's Law:

When an ordinary light is incident on the surface of transparent medium, the reflected light is completely plane polarized and the transmitted light is partially plane polarized. The degree of polarization depends upon the angle of incidence. For a particular angle of incidence, the reflected light is completely polarized. The angle of incidence is known as angle of polarization or Brewster's angle.

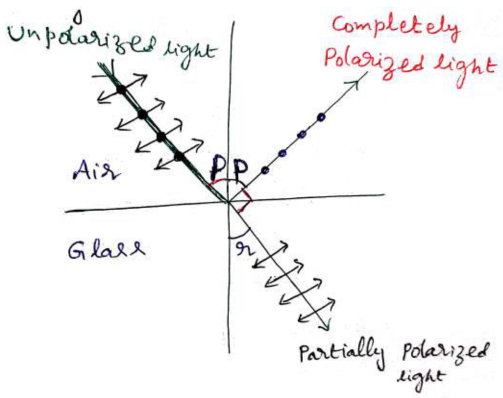


Fig: Polarization by reflection

According to Brewster "the tangent of the angle of polarization for a given medium is equal to the refractive index of that medium. [i.e., $\mu = \tan p$] and at the angle of polarization; the reflected rays and the refracted rays are perpendicular to each other." This is called Brewster's law.

$$\mu = \tan p$$

$$= \frac{\sin p}{\cos p} \quad ....(1)$$

From Snell's law,

$$\mu = \frac{\sin i}{\sin r} \quad .... \quad (2)$$

$$= \frac{\sin p}{\sin r} \quad (\because p = i)$$

Equating (1) and (2),

$$\frac{\sin p}{\cos p} = \frac{\sin p}{\sin r}$$

$$\sin r = \cos p$$

$$\cos\left(90^\circ - r\right) = \cos p$$

$$90^\circ - r = p$$

$$\angle p + \angle r = 90^\circ$$

Thus, at the angle of polarization, the reflected and refracted rays are perpendicular to each other.

Polarization by Double refraction or Birefringence:

When a ray of unpolarized light incident on anisotropic crystals such as calcite or quartz, it is refracted in two rays. One of these rays obeys the ordinary laws of refraction and having vibrations perpendicular to the principal section or plane of the crystal is called ordinary ray while the ray that generally does not obeys the laws of refraction and vibrations parallel to the principal section of the crystal is called extraordinary ray. Both the rays are plane polarized. This phenomenon is called double refraction.

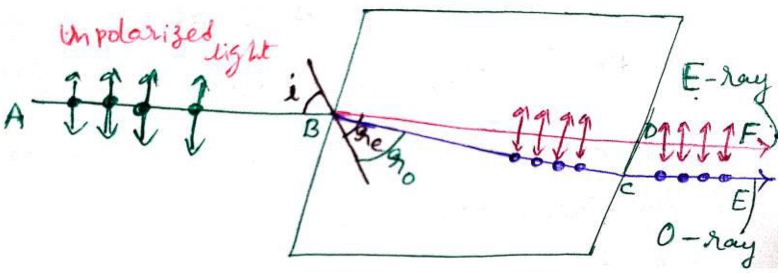


Fig.**:** Polarization by double refraction

Consider AB be an incident unpolarized ray at the surface of calcite crystal. It undergoes double refraction. Ordinary ray travels along BC and extra ordinary ray travels along BD. r_e and r_0 are the angles of refractions for ordinary and extra ordinary rays and finally they emerge as parallel rays along CE and DF.

The refractive index of ordinary ray is μ_0. $\mu_0 = \frac{\sin i}{\sin r_0}$

And the refractive index of extra ordinary ray is, $\mu_e = \frac{\sin i}{\sin r_e}$

For calcite, $r_0 < r_e$ so, $\mu_0 > \mu_e \Rightarrow v_e > v_0$

Here, the velocity of ordinary rays through the crystal is less than that of the extra ordinary ray. μ_0 is same for all the angles of incidence while μ_e varies with the angle of incidence. Ordinary ray travels with same velocity in all directions while extra ordinary ray travels with different speeds in different directions.

Huygens theory of double refraction:

Huygens's theory states that every point on a wave front acts as a secondary source of wavelets. The new wave front is envelope of secondary wavelets.

According to this theory:

i) When any wave front strikes a double refracting crystal, every point of crystal becomes a source of two wave fronts.

(a) O-wave front corresponds to ordinary ray, since ordinary ray has same velocity in all directions. The ordinary wave front is spherical in shape.
(b) E-wave front corresponds to extra ordinary ray, since extra ordinary ray has different velocities in different directions. The extra ordinary wave front is ellipsoidal in shape.

ii) In certain crystals called negative crystals, the ellipsoid lies outside the sphere as shown in the figure. Thus, in negative crystals the extra ordinary wave front travels faster than the ordinary wave front except along optic axis.

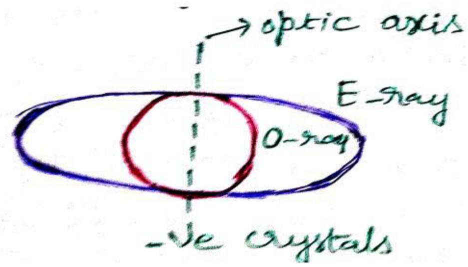


Fig: Representation of negative crystals

iii) In certain crystals called positive crystals, the ellipsoid lies inside the sphere as shown in the figure. Thus, in positive crystals the ordinary wave front travels faster than the extra ordinary wave front except along optic axis.

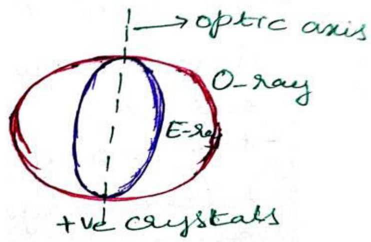


Fig: Representation of positive crystals

Nicol Prism: Nicol prism is an optical device used for producing and analyzing the plane polarized light. It was invented by ***William Nicol*** in 1828 and is known as Nicol prism. It works based on the ***principle of double refraction***, in which the ordinary ray is cut off by total internal reflection, while the extra ordinary ray is allowed to pass through it.

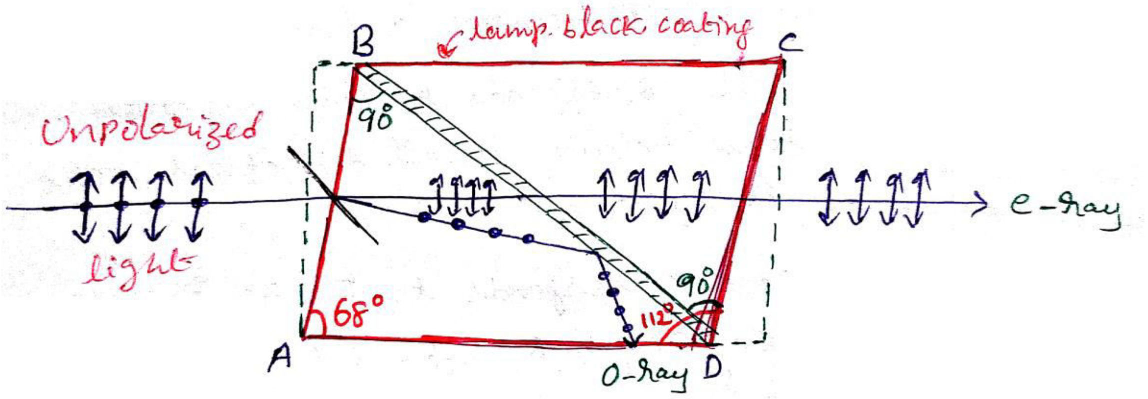
Construction:

It is constructed from calcite crystal whose length is three times its breadth. The end faces $AB \& CD$ of this crystal are grounded in such way that the angles in the principal plane or principal section becomes 68° and 112° instead of 71° and 109°. This is done to increase the field of view. The crystal is now cut along the diagonal BD into two pieces by a plane perpendicular to the principle section as well as end faces AB and CD as shown in figure. The cut faces are grounded and polished optically flat and then again cemented together by Canada balsam, which is a transparent material. It is optically denser than calcite for e-ray but less dense for o-ray. The sides of BC and AD are coated with lamp black.

For sodium light; Refractive index for O- ray $\mu_0 = 1.6584$

Refractive index for e-ray $\mu_e = 1.4864$

Refractive index for Canada balsam $\mu = 1.55$



Action or Working of Nicol Prism:
When a ray of unpolarized light is incident on end face AB of the crystal, it is divided into two components o-ray and e-ray. When the o-ray reaches Canada balsam layer it passes from optically denser medium to rarer medium. As the length of the crystal is large, this ray is made to strike the Canada balsam layer at an angle more than critical angle of 69°. Hence, it is total internally reflected and it is absorbed by black end walls. The e-ray freely passes and emerges out of the crystal as it passes from optically rarer medium to denser medium. Thus, an intense beam of polarized light emerges out with vibration parallel to the principle section.
Uses or applications of Nicol prism:
Two Nicol's prisms are lined up one behind the other is used in optical microscopes for studying optical properties of light. When two Nicol prisms are arranged co-axially as shown in the figure, then the first Nicol which produces polarized light is called polarizer where as the second Nicol that analyses the polarized light is known as analyzer.
In parallel position the light from the polarizer passes through analyzer as shown in figure (a), upon the analyzer rotate through 90° no light is transmitted. In this case the extraordinary light in the second Nicol is totally reflected as shown in figure (b).

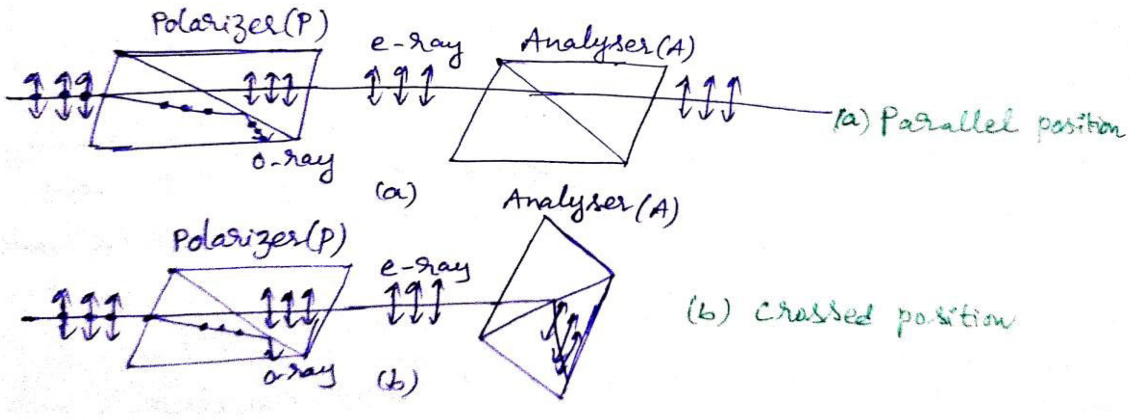


Fig: Nicol Prism as polarizer and analyzer
Quarter Wave Plate: "A quarter wave plate is a plate cut from the doubly refracting crystal of proper thickness parallel to its optic axis and is employed to introduce a phase difference of $90^\circ\left(\frac{\pi}{2}\right)$ or path difference of $\frac{\lambda}{4}$ between the ordinary ray and extra ordinary ray on transmitting light normally through it".

The ordinary ray and extra ordinary ray travel along the same direction but with different velocities. Let t be the thickness of the crystal plate then the optical path of o-ray is $\mu_0 t$

Optical path of e-ray is $\mu_e t$

Hence the optical path difference is $(\mu_0 \sim \mu_e)t$

The optical path difference is equal to $\frac{\lambda}{4}$.

$$(\mu_0 \sim \mu_e)t = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{4(\mu_0 - \mu_e)} \quad (\text{for negative cyrstals})$$

Here, μ_0 is the refractive index of ordinary ray and μ_e is the refractive index of extra ordinary ray.

$$(\mu_e - \mu_0)t = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{4(\mu_e - \mu_0)} \quad (\text{for positive cyrstals})$$

This is used to produce circularly and elliptically polarized light.

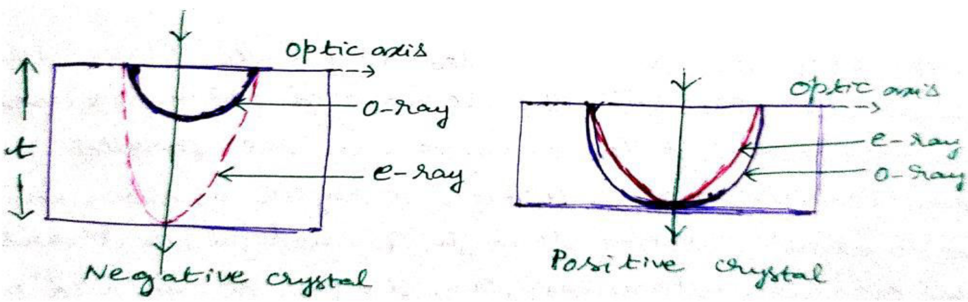


Fig : Representation of positive crystal and negative crystal

Half wave plate: "A half wave plate is a plate cut from the doubly refracting crystal of proper thickness parallel to its optic axis and is employed to introduce a phase difference of $180^\circ(\pi)$ or path difference of $\frac{\lambda}{2}$ between the ordinary ray and extra ordinary ray on transmitting light normally through it."

The ordinary ray and extra ordinary ray travel along the same direction but with different velocities. Let t be the thickness of the crystal plate then the optical path of o-ray is $\mu_0 t$

Optical path of e-ray is $\mu_e t$

Hence the optical path difference is $(\mu_0 \sim \mu_e)t$

The optical path difference is equal to $\frac{\lambda}{2}$.

$$(\mu_0 \sim \mu_e)t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{2(\mu_0 - \mu_e)} \quad (\text{for negative cyrstals})$$

Here, μ_0 is the refractive index of ordinary ray and μ_e is the refractive index of extra ordinary ray.

$$(\mu_e \sim \mu_0)t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{2(\mu_e - \mu_0)} \quad (\text{for positive cyrstals})$$

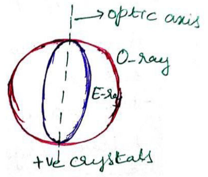
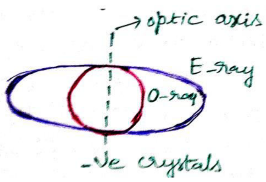
***This is used to produce plane polarized light and it is used as half shade device in a polarimeter*.**

Differences between Ordinary light ray (O-ray) & Extra Ordinary light ray (E-ray):

S.No	*Ordinary light ray(O-ray)*	*Extra Ordinary light ray(E-ray)*
1	It obeys the laws of refraction and hence its refractive index is constant and does not change with the direction of propagation.	It does not obey the laws of refraction, and hence its refractive index is not constant and varies with the angle of incidence as well as direction of propagation.
2	It obeys the Snell's law.	It does not obey the Snell's law
3	For calcite crystal, the refractive index of O-ray is μ_0 = 1.6584 (for sodium yellow light) and it is constant in all directions.	The refractive index of E-ray along the direction of optic axis is μ_e = 1.6584 and normal to the direction of optical axis is μ_e =1.4864.
4	The vibration of the electric vector corresponding to the O-ray is always at right angles to the direction of E-ray as well as to the optic axis.	The vibration of the electric vector corresponding to the E-ray is at right angle to the propagation vector K and lies in the plane containing optic axis and the ray direction.
5	The velocity of O-ray inside the negative (calcites) crystals is less than the E-ray because the angle of refraction is less; hence the refractive index is more.	The velocity of E-ay inside the negative (calcite) crystals is more than O-ray because the angle of refraction is more; hence the refractive index is less.
6	It travels with less spend inside the calcite crystal and its velocity is same in all directions.	It travels faster than O-ray inside the calcite crystal and its velocity changes with direction.
7	For positive crystals like quartz crystals, the refractive index is less than E-ray.	For positive crystals, the refractive index is more than O-ray.
8	The velocity of O-ray in the positive crystals is more than E-ray.	In positive crystals, the velocity of this ray is smaller than O-ray.

Differences between positive and negative crystals:

S.No	*Positive Crystals*	*Negative Crystals*
1	If the elliptical wave front lies completely inside the spherical wave front then the crystal is known as positive crystal.	If the elliptical wave front lies completely outside the spherical wave front then the crystal is known as negative crystal
2	Quartz is the example for positive crystal.	Calcite is the example for negative crystal
3	The velocity of O-ray in the positive crystals is more than E-ray.	The velocity of O-ray in the negative crystals is less than E-ray.
4	For positive crystals the refractive index is less for E-ray	For negative crystals the refractive index is more for E-ray
5	The velocity of E-ray inside the positive crystals is less than the O-ray	The velocity of E-ray inside the negative crystals is more than the O-ray
6	The angle of refraction for O-ray is less	The angle of refraction for O-ray is more
7	The angle of refraction for E-ray is more	The angle of refraction E-ray is less
8	The refractive index for O-ray is more.	The refractive index for O-ray is less.
9	The refractive index for E-ray is less.	The refractive index for E-ray is more.

10	In positive crystals the ordinary wave front travels faster than the extra ordinary wave front except along optic axis.	In negative crystals the extra ordinary wave front travels faster than the ordinary wave front except along optic axis
11	For positive crystals $r_0 > r_e$ so, $\mu_0 < \mu_e \Rightarrow v_e < v_0$	For negative crystals $r_0 < r_e$ so, $\mu_0 > \mu_e \Rightarrow v_e > v_0$
12		

Numerical Problems

1. Find the polarizing angle at a refractive index of 1.732

Sol: Given:

The refractive index of glass is, $\mu = 1.732$

According to Brewster's law, $\mu = \tan i_p$

The polarizing angle of glass is,

$$\mu = \tan i_p$$

$$i_p = \tan^{-1}(\mu) = \tan^{-1}(1.732) = 60^\circ$$

2. Find the thickness of the half wave plate, when the wavelength of light is equal to $5890\ \text{A}^\circ$ **(**$\mu_0 = 1.55\ \ \mu_e = 1.54$**)**

Sol: Given: The wavelength of light,= $\lambda = 5890\ \text{A}^\circ = 5890 \times 10^{-10}$ m

The refractive index of ordinary ray, $\mu_0 = 1.55$

The refractive index of extraordinary ray, $\mu_e = 1.54$

We know that the thickness of the half wave plate is

$$t = \frac{\lambda}{2(\mu_e - \mu_0)} = \frac{5890 \times 10^{-10}}{2(1.55 - 1.54)} = 29.45 \times 10^{-6}\,\text{m} = 29.45\ \mu\text{m}$$

The thickness of half wave plate, $t = 29.45\ \mu$m

4. **Calculate the thickness of a quarter wave plate for light of wavelength** $6000\ \text{A}^\circ$ $\mu_0 = 1.554$ **and** $\mu_e = 1.544$

Sol: Given: The wavelength of light,= $\lambda = 6000\ \text{A}^\circ = 6000 \times 10^{-10}$ m

The refractive index of ordinary ray, $\mu_0 = 1.554$

The refractive index of extraordinary ray, $\mu_e = 1.544$

We know that the thickness of the quarter wave plate is,

$$t = \frac{\lambda}{4(\mu_e - \mu_0)} = \frac{6000 \times 10^{-10}}{4(1.554 - 1.544)} = 1.5 \times 10^{-5}\,\text{m}$$

The thickness of quarter wave plate, $t = 1.5 \times 10^{-5}$ m

Q. No		UNIT-I Assignment questions	Marks	CO	RBT
1	A	Explain the principle of superposition of waves with necessary diagrams and theory?	7	CO1	Understand
	B	Calculate the Brewster angle for ethanol for which μ = 1.45	3	CO1	Apply
2	A	Derive the condition for intensity of bright and dark interference fringes by Cosine Law in optics.	7	CO1	Understand
	B	Calculate the thickness of a half wave plate of quartz for a wave length 500nm. Here μ = 1.553 and μ = 1.544	3	CO1	Apply
3	A	Describe the Brewster's law and Explain how it can be used to produce plane polarized light.	7	CO1	Understand
	B	A parallel beam of sodium light of wavelength 5893A° is incident at an angle of 45° on a film of mustered oil (μ=2.6) on water. Calculate the smallest thickness of the film which will make it appear dark?	3	CO1	Apply
4	A	Explicate the interference phenomenon observed in Newton's Rings and derive the conditions for the diameters of the bright and dark circular rings.	10	CO1	Understand
5	A	Discuss the construction and working of a Nicol's prism with neat diagrams.	8	CO1	Understand
	B	Newton's rings are observed in the reflected light of wavelength 5900A°. The diameter of the 10^{th} dark ring is 0.5 cm. Find the radius of curvature of lens used.	2	CO1	Apply
6	A	Explain in detail the Fraunhoffer diffraction due to single slit with neat intensity curves.	10	CO1	Understand
7	A	Illustrate your explanation with the necessary diagrams and theory of Fraunhoffer diffraction due to a double slit.	10	CO1	Understand
8	A	Describe the theory behind Fraunhoffer diffraction due to 'N' slits, and explain how the number of slits influences the diffraction pattern.	8	CO1	Understand
	B	For a glass plate of refractive index μ = 1.54, calculate the angle of polarization and also angle of refraction.	2	CO1	Apply
9	A	What are quarter & Half wave plates? Derive the expressions for the thickness of quarter & half wave plates	7	CO1	Understand
	B	In Newton's ring experiment the diameter of the 15^{th} dark ring was found to be 0.590cm and that of the 5^{th} dark ring 0.336cm. If the wavelength of the light used is 6000 A^0 calculate the radius of the Plano convex lens	3	CO1	Apply
10	A	Describe the diffraction grating Spectrum and deduce an expression for the wavelength of light with neat diagram. And also narrate its characteristics.	7	CO1	Understand
	B	A half wave plate is designed from a crystal for λ =600nm. If $(\mu_o-\mu_e)$ = 0.0057, calculate the thickness of the plate.	3	CO1	Apply

S.No	**Numerical Problems on Unit-1**	M
1	Calculate the Brewster angle for ethanol for which μ = 1.45	3
2	Calculate the thickness of a half wave plate of quartz for a wave length 500nm. Here μ = 1.553 and μ = 1.544	3
3	A parallel beam of sodium light of wavelength 5893A° is incident at an angle of 45° on a film of mustered oil (μ=2.6) on water. Calculate the smallest thickness of the film which will make it appear dark?	3
4	Newton's rings are observed in the reflected light of wavelength 5900A°. The diameter of the 10^{th} dark ring is 0.5 cm. Find the radius of curvature of lens used.	3
5	For a glass plate of refractive index μ = 1.54, calculate the angle of polarization and also angle of	3

	refraction.	
6	In Newton's ring experiment the diameter of the 15^{th} dark ring was found to be 0.590cm and that of the 5^{th} dark ring 0.336cm. If the wavelength of the light used is 6000 A^0 calculate the radius of the Plano convex lens	3
7	A half wave plate is designed from a crystal for λ =600nm. If $(\mu_o-\mu_e)$ = 0.0057, calculate the thickness of the plate.	3
8	A plane transmission grating having 4250 lines/cm is illuminated with sodium light normally. In the second order spectrum the spectral lines deviated by 30^0 are observed. Find the wavelength of spectral line.	3
9	Determine the Brewster angle for alcohol for which μ = 1.369	3

S. No	**Viva questions from Unit-1**	**Marks**
1	Why Newton's rings are circular?	2
2	Enlighten the principle of superposition of waves?	2
3	If instead of monochromatic light white light is used for interference of light, what would be the change in the observation?	2
4	Write any two conditions for sustained interference?	2
5	Zero order Interference fringe can be identified by which light?	2
6	Explain the phenomenon of diffraction?	2
7	Describe the dispersive power of diffraction grating?	2
8	Distinguish between Fraunhofer's and Fresnel's diffraction?	2
9	Sketch the grating spectrum for monochromatic light & label it.	2
10	A plane transmission grating having 4250 lines/cm is illuminated with sodium light normally. In the second order spectrum the spectral lines deviated by 30^0 are observed. Find the wavelength of spectral line.	2
11	Elucidate the Brewster's law in optics?	2
12	Differentiate polarized and unpolarized light?	2
13	Compare e-ray and o-ray?	2
14	Outline positive and negative crystals with one example each?	2
15	Determine the Brewster angle for alcohol for which μ = 1.369	2